SOME PROBLEMS OF THE DYNAMICS OF VAPOR BUBBLES UNDER CONDITIONS OF A WEAK BODY-FORCE FIELD

Yu. A. Kirichenko

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We determine the reaction force of a liquid acting on a vapor or gas bubble which expands and rises from a horizontal wall. Relations are presented for the radii of bubbles at breakaway for small Jacob numbers. We explain the weak dependence of heat-transfer coefficients on gravitational acceleration in developed bubble boiling.

The kinetics of formation and the dynamics of growth and separation of vapor bubbles are amongst the most important problems in the physics of boiling. Recently the development of studies of heat transfer under conditions of weak body-force fields has offorded a new opportunity for the evaluation of various relationships relating to boiling. Papers on the dynamics of vapor bubbles under these conditions are few in number [1-4] and were carried out in the overload range, $0.01 < \eta < 1$.

The studies [1, 2] were carried out on water using flat nickel heaters under conditions of short-term "weightlessness," achieved at incidence of the container with the mounting.

The results given in [3, 4] were obtained under conditions of long-term simulation of weak body-force fields [5, 6] during the boiling of ethyl ether on a belt heater [4] and of oxygen at a single nucleus of boiling. Studies of the boiling of water [2] and of ethyl ether [4] were carried out on polished surfaces containing a limited number of vapor-forming nuclei. Thus the majority of the results obtained apply to individual bubbles; quite obviously, the concepts devised for describing an ensemble of interacting bubbles do not apply to them (see, for example, [7, 8]).

The main results obtained in [1-4] reduce to the following:

1. The dependence of the bubble radius on the time in the asymptotic stage of bubble growth may be expressed by the relation

$$R = \beta\tau^n, \tag{1}$$

where $n \approx 0.4$-0.5;

$$\beta = C_0 \,\mathrm{Ja}\sqrt{a} = C_0 \frac{\lambda \Delta T}{\rho'' L a^{1/2}} \quad (\text{for} \quad n = 0.5); \tag{2}$$

C_0 is a coefficient on the order of unity. The values of β and n are very close to the theoretical values [9] and are independent of g.

2. During breakaway the bubble radii increase as g decreases in accordance with the law

$$R_d = A_1 g^{k_1}, \tag{3}$$

where $k_1 \approx -(0.3$-$0.4)$.

3. The breakaway frequency of the bubbles is described by the relationship

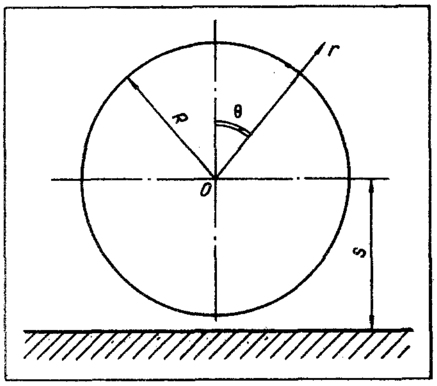

Fig. 1. Scheme for calculation of bubble motion near the wall.

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$$f = A_2 g^{k_2},\tag{4}$$

where $k_2 \approx 0.7$-1.

4. The ascent speed of the "largest" bubbles (10^{-3} m < R_d < $3 \cdot 10^{-3}$ m, for $\eta = 1$) is expressed by the relationship [2]

$$u = A_3 \sqrt[4]{\frac{\sigma g(\rho - \rho'')}{\rho^2}}.\tag{5}$$

The ascent speed of the "small" bubbles ($2 \cdot 10^{-4}$ m < R_d < 10^{-3} m for $\eta = 1$) [3, 4] is in accord with the relationship [10]

$$u = A_4 \sqrt{R_d g},\tag{6}$$

which, with $k_1 = -1/3$ in Eq. (3), corresponds to $u \sim g^{1/3}$.

5. For the Jacob numbers used in [1-4] (Ja ≈ 10-30) bubble breakaway started from a "neck," the bubble speed at the time of its departure from the "neck" being close to its speed far from the surface.

One of the most interesting of our results is the dependence of R_d on g. From a perusal of [2, 3] it follows that the known relationship due to Fritz [11]

$$R_d \approx 0.01\gamma \sqrt{\frac{\sigma}{g(\rho - \rho'')}}\tag{7}$$

is not satisfied even for very small Jacob numbers (Ja ≈ 10-20). For cryogenic liquids with wetting angles with metals $\gamma \approx 0°$, the relationship (7) is simply not applicable. Obviously, for a true estimate of R_d it is necessary to account for the dynamics of bubble growth and not be restricted to a study of static stability of the bubble.

We estimate the dynamic forces (reaction forces of the liquid) acting from the side of the liquid on a vapor bubble as it grows in size and ascends from the wall. Assuming a spherical bubble, taking the flow of the liquid to be a potential flow, and neglecting the momentum of the vapor, we find that the velocity potential φ satisfies Laplace's equation

$$\Delta\varphi = 0\tag{8}$$

with corresponding boundary conditions [12]. The potential for a bubble far from the wall for axially symmetric flow of the liquid [13] is given by

$$\varphi = \frac{R^3}{2r^2}\dot{s}\cos\theta + \frac{R^2}{r}\dot{R}.\tag{9}$$

The meaning of the quantities r, R, s, and θ is shown in Fig. 1. Using the method of images [12] we can obtain an expression for φ taking into account the influence of the wall, in the form of a power series [13]; after simple transformations, this expression reduces to the form

$$\varphi = \dot{s}\left[\frac{R^3}{2r^2} + \left(\frac{R^3}{2r^2} + r\right)\frac{R^3}{8s^3 - R^3}\right]\cos\theta + \dot{R}\left[\frac{R^2}{r} + \left(\frac{R^3}{2r^2} + r\right)\frac{2R^2 s}{8s^3 - R^3}\cos\theta\right].\tag{10}$$

Substituting Eq. (10) into Bernoulli's equation [12]

$$\frac{p}{\rho} = \frac{\partial\varphi}{\partial\tau} - \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial r}\right)^2 + \left(\frac{1}{r}\cdot\frac{\partial\varphi}{\partial\theta}\right)^2\right] - g(s + r\cos\theta) + \frac{P_\infty}{\rho}\tag{11}$$

and integrating Eq. (11) for the pressure over the surface of the bubble, we obtain the total force F acting on the vapor bubble:

$$F = F_R - F_g.\tag{12}$$

Here F_g is the buoyancy force:

$$F_g = \frac{4}{3}\pi R^3 \rho g,\tag{13}$$

F_R is the reaction force:

$$\frac{F_R}{\frac{4}{3}\pi\rho R^2} = \frac{1 + 2\varkappa^3}{2(1 - \varkappa^3)}\ddot{s}R - \frac{9\varkappa^4}{(1 - \varkappa^3)^2}\dot{s}^2 + \frac{3}{2}\cdot\frac{1 - 2\varkappa^3 - 2\varkappa^6}{(1 - \varkappa^3)^2}\dot{s}\dot{R} + \frac{9}{2}\cdot\frac{\varkappa^2}{(1 - \varkappa^3)^2}\dot{R}^2 + \frac{3}{2}\cdot\frac{\varkappa^2}{1 - \varkappa^3}\ddot{R}R,\tag{14}$$

where $\varkappa = R/2s$.

TABLE 1. Data on Vapor Bubble Dynamics from [1-4]

Lit. data	$\beta\cdot 10^2$, m/sec$^{1/2}$	$R_d\cdot 10^3$, m (experiment)	$K_{Rg}\cdot 10^2$	$R_d\cdot 10^3$, m (theory)
[1]	1,35	3	2	2,1
[2]	0,8	1,33	4	1,26
[3]	0,235	0,24	6	0,27
[4]	0,42	0,6	4	0,57

For the case of a bubble tangent to the wall (R = s), we have for the velocity potential, instead of Eq. (10),

$$\varphi = \dot{R}\left(\frac{5}{7}\cdot\frac{R^3}{r^2}+\frac{3}{7}r\right)\cos\theta+\frac{\dot{R}R^2}{r}. \tag{15}$$

The expression for the reaction force in this case reduces to

$$F_R = \frac{4}{3}\pi R^2\rho\left(\frac{8}{7}\ddot{R}R+\frac{15}{7}\dot{R}^2\right). \tag{16}$$

Equations (14) and (16) have the same structure as the approximate expressions obtained in [13] and agree with them as to order of magnitude.

From Eqs. (1) and (16) we can obtain the following expression for the reaction force:

$$F_R = \frac{4\pi}{21}\rho\beta^4 n(23n-8)\tau^{4n-2}. \tag{17}$$

From Eq. (17) it is evident that when n = 8/23 the dynamic force is equal to zero; for n < 8/23, it is negative (directed upwards from the wall); for n > 8/23, it is positive (directed towards the wall); for n = 1/2, the force is constant; for n < 1/2, it decreases with time; for n > 1/2, it increases with time. The elementary analysis given here shows the inaccuracy of general statements regarding the single-valued action of the dynamic forces (inertia forces) on a vapor bubble.

Equation (17) agrees to within around 10% with the expression for F_R obtained in [14] for n = 1/2 by another method. From Eqs. (1), (12), and (16) we can obtain a criterion for estimating the size of the dynamic forces in relation to the buoyancy force

$$K_{Rg} = \frac{F_R}{F_g} = \frac{\beta^{\frac{2}{n}} R^{1-\frac{2}{n}} n(23n-8)}{7g}. \tag{18}$$

We can take the condition $K_{Rg} = 1$ as an estimate of the minimum bubble departure radius (for n > 8/23), since in this case forces of surface tension are not considered. Thus

$$R_{d\min} = \beta\left[\frac{\beta n(23n-8)}{7g}\right]^{\frac{n}{2-n}}. \tag{19}$$

From Eq. (19), using Eq. (2) we obtain as a special case, when n = 1/2, expressions analogous to those of Ruckenstein [15], and Roll and Myers [16], with a value for the exponent of g equal to −1/3. In general, this exponent assumes various values, for example, from −1/4 to −2/3, when n varies from 0.4 to 0.9.

Estimates K_{Rg} for vapor bubbles at the instant of departure (see Table 1) show that the dynamic forces (reaction forces) acting on a bubble from the side of the liquid are small. In view of the smallness of the dynamic forces at the instant of bubble breakaway, an attempt to estimate the breakaway radius from a formal consideration of the equilibrium of forces is not feasible. The expression (19), obtained by equating the dynamic and buoyancy forces, yields lowered results. For example, in [3], when $\eta = 1$, $R_{d\,\min} \approx 0.9\cdot 10^{-4}$ m; the experimental results yield $R_d \approx (1.5\text{-}3)\cdot 10^{-4}$ m.

We consider now the equilibrium of forces, taking surface tension into account [11]:

$$F_g = F_\sigma + F_R, \tag{20}$$

where $F_\sigma = 2\pi R\sigma\varphi(\gamma)$.

From Eq. (20) we obtain for n = 1/2 the relation

TABLE 2. Dependence of Quantities Characterizing Vapor Bubble Breakaway on Gravitational Acceleration

Bubble ascent speed	n	Exponent of g				
		R_d	f	fR_d	fR_d^2	fR_d^3
$\dot{s} \sim \sqrt{Rg}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$
	$\frac{2}{5}$	$-\frac{1}{4}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$-\frac{1}{8}$
$\dot{s} \sim g^{1/4}$	$\frac{1}{2}$	$-\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{8}$	0	$-\frac{1}{8}$
	$\frac{2}{5}$	$-\frac{3}{10}$	$\frac{3}{4}$	$\frac{9}{20}$	$\frac{3}{20}$	$-\frac{3}{20}$

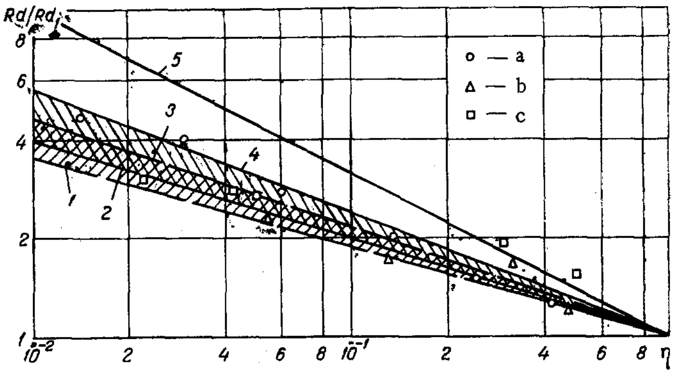

Fig. 2. Relative radii of vapor bubbles as a function of gravitational acceleration: a) data of [1]; b) data of [2]; c) data of [3]; 1) $k_1 = -1/4$; 2) $-3/10$; 3) $-1/3$; 4) $-3/8$; 5) $-1/2$. The lower crosshatched region refers to [3]; the upper to [1, 2].

$$R \approx R^* \left(1 + \frac{\beta^2}{4gR^{*2}} \dot{R}\right), \tag{21}$$

where R^* is the breakaway radius according to Fritz (7).

Equation (21) is the analog of Stanishevskii's formula, referred to by the authors of [2] in interpreting their data. However, in accordance with the data of [2], the coefficient of $\dot{R}$ in Eq. (21), being independent of g since $R^* \sim g^{-1/2}$, has a value of the order of 1 sec/m when $\dot{R} \approx 10^{-2}$ m/sec for $\eta = 1$. Thus the procedure of estimating the influence of bubble-growth dynamics on the breakaway of the bubble from the wall by using equations of the type of Eq. (20) cannot be considered to be correct.

Let us attempt to estimate the bubble breakaway radius from the conditions of equilibrium of forces acting on the bubble, not when the bubble is isolated from the surface, but at the break in the "neck" of the bubble where surface tension cannot be taken into account. At this instant the shape of the bubble is closest to spherical [3], testifying thereby to the absence of forces deforming the bubble and permitting us to apply, more or less correctly, the theory outlined below which is applicable to a spherical bubble.

Neglecting, for simplicity, the influence of the wall, i.e., putting $\varkappa = 0$ in Eq. (14), we obtain

$$F_R = \frac{2}{3} \pi R^2 \rho (3\dot{R}\dot{s} + R\ddot{s}). \tag{22}$$

Putting $F_g = F_R$, and noting from Eqs. (22), (1), and (13) that $\dot{s} = u = \text{const}$, and $\ddot{s} = 0$, we have

$$R_d = \left(\frac{3n}{2}\right)^n \beta g^{-n} \dot{s}^n, \tag{23}$$

where $\dot{s} = u$ in accord with Eq. (5) or Eq. (6).

In Table 2 we present information giving the dependence of the breakaway radius R_d on g; we also give the dependence on g of the breakaway frequency f, as well as the quantities fR_d, fR_d^2, and fR_d^3, calculated on the basis of Eq. (23).

From Table 2 it follows, in particular, that the exponent k_1 in Eq. (3) depends on the size of the bubbles and on the quantity n in Eq. (1). For the cases of boiling studied in [1-4], with Jacob number Ja ≈ 10 to 30, this exponent may vary from $-1/4$ to $-3/8$, which is very close to the experimental values. The good agreement of the experimental and theoretical results in evident in Fig. 2.

To obtain expressions for the absolute magnitudes of the bubble departure radii we put $\dot{s} = u$ from Eqs. (5) and (6) into Eq. (23), revising thereby the coefficients A_3 and A_4 from the experimental data [2, 4]. For small bubbles ($2 \cdot 10^{-4}$ m < R_d < 10^{-3} m for $\eta = 1$)

$$R_d = 1.85\beta^{4/3} g^{-1/3} \tag{24}$$

and for "large" bubbles (10^{-3} m < R_d < $3 \cdot 10^{-3}$ m for $\eta = 1$)

$$R_d = 1.2\beta \left(\frac{\sigma}{\rho}\right)^{1/8} g^{-3/8}. \tag{25}$$

In evaluating Eqs. (24) and (25), for definiteness, we took $n = 1/2$. The values of R_d, calculated in accord with Eqs. (24) and (25), agree with the experimental values from [2-4] to within several percent (see Table 1).

Equation (25) also helps to explain the very weak dependence of the heat-transfer coefficient α on g, which was observed experimentally for $\eta = 0.01$-1 for developed bubble boiling [1, 3, 5, 6, 17]. Assuming that all the heat from the heating surface goes into vapor formation, we obtain

$$q = \frac{4}{3} \pi R_d^3 f L \rho'' N. \tag{26}$$

Using Eqs. (2) and (24), we write Eq. (26) in the form

$$\alpha = B_1 \frac{N^{0.3} \lambda}{L^{0.3} \rho''^{0.7} a^{0.5} g^{0.1}} q^{0.7}, \tag{27}$$

where $B_1 \approx 2$.

If we use Zhokhov's development for N [18], we obtain from Eq. (26)

$$\alpha = B_2 \frac{l^{3/19} \lambda^{10/19} \rho''^{2/19} L^{2/19}}{a^{5/19} T_s^{9/19} \sigma^{9/19} g^{1/19}} q^{16/19}, \tag{28}$$

where B_2 is a dimensionless coefficient; the quantity $l = 1$ m has been introduced to maintain the proper dimensionality [18].

Equations (27) and (28) furnish a plausible dependence of α on q and g, and, in addition, Equation (28) furnishes a qualitatively correct dependence of α on the pressure. It is interesting to note that in the majority of the experimental papers on boiling in the presence of decreased gravitation [17], a very weak increase in α was obtained with a decrease in g, a result which also follows from Eqs. (27) and (28).

NOTATION

$\eta = g/g_1$	is the overload coefficient;
g	is the gravitational acceleration;
g_1	is the Earth's gravitational acceleration;
R	is the radius of the bubble;
R_d	is the breakaway radius of the bubble;
R_{d_1}	is the bubble breakaway radius at $\eta = 1$;
$R_{d\,min}$	is the minimum breakaway radius;
$Ja = c\rho\Delta T / L\rho''$	is the Jacob number;
τ	is the time;
γ	is the wetting angle, deg;
c	is the specific heat at constant pressure;
ρ	is the density of the liquid;
ρ''	is the density of the vapor;

L	is the latent heat of vaporization;
ΔT	is the temperature difference between the heater and the bulk of the liquid;
λ	is the thermal conductivity;
a	is the thermal diffusivity;
σ	is the surface tension;
u	is the lift velocity of the bubble from the surface;
θ, r	are the coordinates of a point in the coordinate system having its origin at the center of the bubble;
φ	is the velocity potential;
s	is the distance from the center of the bubble to the wall;
p	is the pressure;
T_s	is the saturation temperature;
N	is the number of centers of vapor generation per unit area;
α	is the heat-exchange coefficient;
q	is the heat flux density.

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